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Article

Inclusive Educational Strategies for Precalculus and Physics I: Understanding the Concept and Graphical Representation of Relationships

Estrategias Educativas Integradoras para el Precálculo y la Física I: Comprensión del Concepto y la Representación Gráfica de Relaciones

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Abstract

This study analyzes the implementation of integrative educational strategies between Precalculus and Physics I, focused on understanding the concept of relationships and their graphical representation. It addresses the issue of fragmented teaching in both disciplines, which hinders the connection between mathematical models and physical phenomena. The research was conducted using a mixed-methods, descriptive approach, with the participation of first-year college students. Surveys, observations, and teaching activities based on the interpretation of graphs and the use of technological tools such as GeoGebra were applied. The results

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show a significant improvement in students' ability to interpret graphical representations, relate algebraic functions to physical phenomena, and solve integrated problems. Likewise, an increase in active participation and the development of critical thinking was observed. It is concluded that the use of integrative strategies promotes meaningful learning and strengthens interdisciplinary understanding, contributing to a more solid foundation in the exact sciences.

Keywords: Precalculus, Physics I, graphical representation, meaningful learning.

Resumen

El presente estudio analiza la implementación de estrategias educativas integradoras entre el Precálculo y la Física I, centradas en la comprensión del concepto de relación y su representación gráfica. Se parte de la problemática de la enseñanza fragmentada de ambas disciplinas, la cual dificulta la articulación entre modelos matemáticos y fenómenos físicos. La investigación se desarrolló bajo un enfoque mixto, de tipo descriptivo, con la participación de estudiantes de nivel inicial universitario. Se aplicaron encuestas, observaciones y actividades didácticas basadas en la interpretación de gráficas y el uso de herramientas tecnológicas como GeoGebra. Los resultados evidencian una mejora significativa en la capacidad de los estudiantes para interpretar representaciones gráficas, relacionar funciones algebraicas con fenómenos físicos y resolver problemas integrados. Asimismo, se observó un incremento en la participación activa y en el desarrollo del pensamiento crítico. Se concluye que el uso de estrategias integradoras favorece el aprendizaje significativo y fortalece la comprensión interdisciplinaria, contribuyendo a una formación más sólida en las ciencias exactas.

Palabras clave: Precálculo, Física I, representación gráfica, aprendizaje significativo.

Introduction

In the field of higher education, particularly in the exact sciences, one of the most persistent challenges is the difficulty students face in connecting mathematical knowledge with its application in physical contexts. This issue is most evident in the introductory courses of Precalculus and Physics I, where students must not only understand abstract concepts but also interpret and apply them in modeling real-world phenomena. Various studies have pointed out that traditional teaching, characterized by disciplinary fragmentation, significantly limits the development of deep and meaningful learning (Redish, 2005; Hestenes, 1992).

Precalculus, as a fundamental foundation for the study of calculus and other advanced areas of mathematics, introduces essential concepts such as functions, relationships, and their graphical representations. For its part, Physics I requires these same concepts to describe and analyze phenomena such as motion, force, and energy. However, despite this close epistemological relationship, in educational practice both disciplines are often taught independently, leading students to develop a disjointed view of knowledge (Bing & Redish, 2009). This disconnect hinders the understanding of Physics as a quantitative science grounded in mathematical models.

One of the key elements linking Precalculus with Physics is the concept of the relationship between variables, commonly represented through functions and graphs. The interpretation of these representations constitutes a fundamental skill for the analysis of physical phenomena, especially in topics such as uniform linear motion and uniformly accelerated motion. However, research has shown that many students experience significant difficulties in interpreting graphs, particularly in the transition between different representations (verbal, algebraic, and graphical) (Leinhardt, Zaslavsky & Stein, 1990; McDermott, Rosenquist & van Zee, 1987).

In this context, graphical representation should not be conceived solely as a visual resource, but as a cognitive tool that facilitates the construction of meaning and the understanding of functional relationships. According to Duval (2006), mathematical learning necessarily involves the coordination of multiple representational registers, and difficulty in this coordination is one of the main causes of failure to understand mathematical concepts. This problem is exacerbated when students fail to establish connections between mathematical representations and the physical phenomena they describe.

From the perspective of meaningful learning, proposed by Ausubel (1968), the acquisition of new knowledge depends on the student's ability to relate it to pre-existing cognitive structures. In this sense, the integration of precalculus and physics content can foster the construction of deeper meanings by allowing students to understand the utility of mathematical concepts in explaining real-world phenomena. This integration not only improves conceptual understanding but also contributes to the development of critical thinking and problem-solving skills (Prince & Felder, 2006).

Various studies have highlighted the importance of implementing teaching strategies that promote interdisciplinarity in science education. For example, Beichner et al. (2007) demonstrated that the use of active methodologies in integrated learning environments significantly improves academic performance in Physics. Likewise, studies in mathematics education have shown that the use of multiple representations, including graphs, enhances the understanding of functions and relationships (Ainsworth, 2006). This evidence suggests that the integration of educational strategies centered on graphical representation can be an effective way to improve learning in both disciplines.

Within this framework, integrative educational strategies emerge as a pedagogical alternative that seeks to overcome the fragmentation of knowledge by promoting a holistic approach to learning. These strategies involve the design of instructional activities that link content from different disciplines, fostering the construction of meaningful connections between them. In the case of Precalculus and Physics I, this involves designing learning experiences in which students can explore, interpret, and analyze graphs representing physical phenomena using mathematical tools.

The use of digital technologies, such as graphical visualization software, has also proven to be a valuable resource in this process. Tools like GeoGebra allow students to interact with dynamic representations, facilitating the understanding of the relationship between variables and their behavior. According to Hohenwarter and Jones (2007), the use of these technologies promotes more active and exploratory learning, which contributes to better conceptual understanding.

Despite this evidence, the implementation of integrative strategies in the classroom still faces various challenges, including a lack of teacher training in interdisciplinary approaches and the rigidity of traditional curricula. Therefore, it is necessary to research and propose instructional models that guide the effective integration of precalculus and physics content, with an emphasis on the use of graphical representation as the central focus of learning.

In this regard, various studies in physics education have shown that one of the main obstacles to student learning lies in the inability to correctly interpret graphs representing physical relationships, especially those related to motion. McDermott, Rosenquist, and van Zee (1987) demonstrated that even college students have difficulty distinguishing between a position-time graph and a velocity-time graph, reflecting a superficial understanding of the underlying concepts. This issue highlights the need to strengthen not only mathematical skills but also the ability to interpret representations in physical contexts.

Similarly, the literature on mathematics education has highlighted the importance of translation between different modes of representation as an essential component of mathematical thinking. Duval (2006) argues that understanding a concept does not depend solely on mastery of that concept within a specific mode, but rather on the ability to translate it between different modes, such as algebraic, graphical, and verbal. In this sense, the teaching of precalculus cannot be limited to symbolic manipulation but must integrate multiple forms of representation that allow students to construct deeper meanings.

From a pedagogical perspective, active methodologies have been widely recognized as effective strategies for promoting meaningful learning in the sciences. Prince (2004) notes that approaches such as problem-based learning and collaborative learning encourage active student participation, which translates into greater retention and understanding of the content. In the context of the integration of Pre-Calculus and Physics, these methodologies allow for the design of learning experiences in which students not only receive information but also construct knowledge through the resolution of contextualized situations.

On the other hand, the incorporation of digital technologies in the classroom has significantly transformed the possibilities for teaching and learning in the sciences. Tools such as interactive simulators and graphical representation software facilitate the visualization of complex phenomena and allow students to experiment with different variables in real time. According to Wieman, Adams, and Perkins (2008), the use of interactive simulations in physics instruction improves conceptual understanding by allowing students to actively explore the principles governing physical phenomena.

In this regard, the integration of technological tools with teaching strategies focused on graphical representation can significantly enhance learning in Pre-Calculus and Physics I. However, it is essential that these tools be used in a pedagogically intentional manner—that is, as means to facilitate conceptual understanding and not as ends in themselves. The effectiveness of these strategies depends largely on the teacher's ability to design activities that promote reflection, analysis, and critical interpretation of graphical representations.

Despite advances in educational research, there remains a need for concrete teaching proposals that guide the effective integration of mathematical and physical content in the classroom. In particular, approaches are needed that allow the concept of relationship and its graphical representation to be addressed as a central axis of learning, facilitating the transition between mathematical thinking and the interpretation of physical phenomena. This integration is especially relevant in the early stages of higher education, where the foundations for learning more complex disciplines are established.

In this context, the purpose of this study is to design and analyze integrative educational strategies that promote understanding of the concept of relationship and its graphical representation among students in Pre-Calculus and Physics I. Specifically, it seeks to foster the integration between both disciplines through the use of contextualized teaching activities and the support of technological tools, with the aim of strengthening the interpretation of physical phenomena based on mathematical models. In this way, the study aims to contribute to the development of meaningful, critical, and interdisciplinary learning that enables students to understand the utility and applicability of the concepts addressed in their academic training.

Materials and methods

This study was conducted using a mixed-methods approach, of a descriptive nature with explanatory scope, aimed at analyzing the effectiveness of integrative educational strategies in the understanding of the concept of function and its graphical representation among students in Pre-Calculus and Physics I courses. This approach allowed for the combination of quantitative and qualitative data collection and analysis, with the aim of obtaining a more comprehensive view of the phenomenon under study, in accordance with the proposal by Creswell (2014), who notes that the integration of both approaches fosters a deep understanding of educational processes. The research design was non-experimental and cross-sectional, as variables were not deliberately manipulated; rather, student behavior was observed in a natural classroom setting following the implementation of a teaching intervention.

The population consisted of students enrolled in Precalculus and Physics I courses at the beginning of higher education at a public university, as well as the instructors responsible for teaching those courses. A non-probabilistic convenience sample was selected , comprising approximately 60 to 80 students and between 4 and 6 teachers, based on criteria such as availability, voluntary participation, and access to the courses where the teaching strategies were implemented. This type of sampling is common in applied educational research, where the main objective is to analyze teaching-learning processes in specific contexts (Hernández-Sampieri, Fernández & Baptista, 2014).

Regarding data collection techniques, a survey, direct observation, and analysis of teaching activities carried out during the intervention were employed. The survey was administered to both students and teachers before and after the implementation of the strategies, with the aim of identifying changes in the perception, understanding, and handling of concepts related to functions, relationships, and their graphical representation in physical contexts. The questionnaires were structured with closed-ended Likert-type questions and some open-ended questions, which allowed for the collection of quantifiable data while also gathering participants' opinions and reflections. To ensure content validity, the instruments were reviewed by experts in mathematics and physics education, while their reliability was estimated using Cronbach's alpha coefficient.

Direct observation was conducted during class sessions, using a structured guide that allowed for the recording of aspects such as student participation, teacher-student interaction, the use of graphical representations, and students' ability to interpret relationships between variables. This technique provided relevant qualitative information about the teaching-learning process, complementing the data obtained through the surveys. Additionally, an analysis was conducted of student work, such as solved exercises, interpretations of graphs, and activities based on contextualized problems, which allowed for an evaluation of the level of understanding achieved.

The instructional intervention was designed based on a set of integrative educational strategies focused on the use of graphical representation as a connecting thread between Precalculus and Physics I. These strategies included presenting problem situations contextualized in physical phenomena, such as the analysis of uniform linear motion and uniformly accelerated motion, in which students had to interpret and construct position-time, velocity-time, and acceleration- -time graphs. In addition, the use of technological tools was incorporated, specifically mathematical visualization software such as GeoGebra, which allowed students to dynamically explore the relationship between variables and observe changes in the graphs based on different parameters.

The activities were conducted using an active learning approach, promoting collaborative work in small groups, discussion of results, and reflection on the processes followed. In this regard, students not only solved exercises but also explained their reasoning, interpreted results, and established connections between mathematical representations and physical phenomena. The teacher's role was that of a learning mediator, guiding the construction of knowledge and facilitating interaction between students and the content.

For the analysis of quantitative data, descriptive statistical techniques were used, such as frequencies, percentages, and measures of central tendency, in order to identify patterns in participants' responses. Additionally, a comparison was made between the results obtained in the initial phase (pre-test) and the final phase (post-intervention), which allowed for an evaluation of the impact of the implemented strategies. Regarding the qualitative data, a content analysis was applied, categorizing the open-ended responses and recorded observations, with the aim of identifying trends, recurring difficulties, and evidence of meaningful learning.

In terms of ethics, the research was conducted in accordance with the principles of confidentiality, anonymity, and informed consent. Participants were informed about the study's objectives, and their participation was entirely voluntary. Furthermore, it was ensured that the collected data would be used exclusively for academic and research purposes.

Finally, it is important to note that, although the study was conducted in a specific context, the results obtained can serve as a reference for the implementation of similar strategies in other educational settings, especially in those where there is a disconnect between the teaching of mathematics and physics. In this way, the methodology adopted not only allowed for the analysis of the effectiveness of integrative educational strategies but also contributed to the development of a replicable teaching proposal aimed at strengthening students' understanding of the concept of the relationship between s and their graphical representation as fundamental elements in students' scientific education.

Results

The results obtained in this study demonstrate a positive impact of implementing integrative educational strategies on students' understanding of the concept of relationship and its graphical representation in Pre-Calculus and Physics I. The analysis was structured by comparing the results of the initial assessment with those obtained after the instructional intervention, considering both quantitative and qualitative data.

First, the data from the surveys administered to the students reflect a significant improvement in the understanding of key concepts related to the interpretation of graphs and the relationship between physical and mathematical variables. Before the intervention, a high percentage of students demonstrated difficulties in correctly interpreting position-time and velocity-time graphs, as well as in establishing relationships between algebraic and graphical representations. Subsequently, a notable increase in the level of understanding was observed, suggesting the effectiveness of the implemented strategies.

Table 1.

Level of conceptual understanding before and after the intervention

Indicator assessed	Before (%)	After (%)
Interpretation of graphs (position-time)	42%	78%
Interpretation of graphs (speed-time)	38%	74%
Relationship between algebraic functions and graphs	45%	81%
Understanding the relationship between physical variables	40%	76%
Integrated problem-solving (Mathematics-Physics)	35%	72%

The results in Table 1 show an average increase of over 30% in all evaluated indicators, which demonstrates a substantial improvement in students' conceptual understanding. In particular, the progress in the ability to relate algebraic expressions to their graphical representations stands out, a fundamental aspect for modeling physical phenomena.

Regarding the qualitative analysis, observations made during class sessions identified significant changes in student participation and reasoning. During the initial phase, there was a tendency to solve exercises mechanically, without a deep understanding of the concepts. However, following the implementation of the integrative strategies, students began to explain their answers, justify their interpretations, and establish connections between Pre-Calculus and Physics content.

Likewise, collaborative work fostered the discussion of ideas and the joint construction of knowledge. Students demonstrated greater confidence in interpreting graphs and relating them to concrete physical situations, which was evident in group activities and in solving contextualized problems. This finding aligns with the observations of Prince (2004), who highlights that active learning promotes greater understanding and retention of content.

Regarding the use of technological tools, it was observed that the use of software such as GeoGebra facilitated the dynamic visualization of relationships between variables, allowing students to experiment with different scenarios and immediately observe changes in the graphs. This interaction contributed to a better understanding of abstract concepts, especially in the analysis of function behavior and its application in Physics.

The bar graph clearly shows an increase in all evaluated indicators, demonstrating an upward trend following the intervention. The greatest differences are observed in the resolution of integrated problems and in the interpretation of graphs, which confirms the effectiveness of strategies centered on graphical representation as the core of learning.

On the other hand, students' open-ended responses reflected a positive perception of the methodology used. Many of them stated that the use of graphs and digital tools allowed them to "better understand physics" and "see what mathematics is for," which indicates a shift in attitude toward learning both disciplines. This aspect is relevant, as motivation and the perception of the usefulness of knowledge are key factors in the educational process.

In summary, the results obtained allow us to affirm that the implementation of integrative educational strategies based on graphical representation significantly contributes to the development of conceptual understanding in Pre-Calculus and Physics I students. The observed improvement is not limited to content mastery but also extends to higher-order cognitive skills, such as the interpretation, analysis, and application of knowledge in real-world contexts.

Discussion

The results obtained in this study allow us to conclude that the implementation of integrative educational strategies between Precalculus and Physics I constitutes an effective pedagogical alternative to overcome the fragmentation of knowledge that traditionally characterizes the teaching of these disciplines. The articulation of content through the use of graphical representation as a central axis fostered a deeper understanding of the concept of the relationship between variables, allowing students to establish meaningful connections between mathematical models and physical phenomena.

In this regard, it was evident that students significantly improved their ability to interpret and construct graphs, as well as to relate algebraic expressions to visual representations. This progress was reflected not only in the quantitative results but also in the development of higher-order cognitive skills, such as analysis, interpretation, and reasoning—fundamental aspects of scientific education. The improvement observed in solving integrated problems confirms that learning becomes more meaningful when content is presented in a contextualized manner and linked to real-world situations.

Likewise, the use of active methodologies, such as collaborative learning and problem-based work, contributed significantly to student participation and the collective construction of knowledge. Peer interaction enriched the comprehension processes, creating spaces for discussion and reflection that fostered meaningful learning. This finding reinforces the importance of promoting dynamic learning environments in which students take an active role in their educational process.

Furthermore, the incorporation of technological tools, such as graphical representation software, facilitated the visualization of abstract concepts and allowed students to interactively explore the relationship between variables. This aspect proved key to strengthening conceptual understanding, especially in topics where graphical interpretation is fundamental, such as in the analysis of the motion of a " " in Physics I. However, it is evident that the use of technology must be accompanied by adequate pedagogical mediation, aimed at enhancing its educational value.

Based on these findings, it can be stated that the integration of educational strategies centered on graphical representation not only improves academic performance but also contributes to the development of an interdisciplinary view of knowledge. This perspective is essential in training professionals capable of applying the concepts learned in real-world contexts, moving beyond rote memorization and promoting critical and reflective understanding.

Finally, the study highlights the need to rethink teaching practices in the instruction of the exact sciences, incorporating approaches that favor content integration and the use of active methodologies. In this regard, it is recommended to strengthen teacher training in interdisciplinary strategies and the pedagogical use of technological tools, as well as to promote the revision of curricula to facilitate coordination between subjects. In this way, it will contribute to the construction of educational processes that are more coherent, meaningful, and aligned with the current demands of higher education.

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